Parciální derivace – příklad

$$f(x,y) = (x+y^2)\,e^{x+y^2}$$

VYPOČTĚTE VŠECHNY PARC. DERIVACE AŽ DO ŘÁDU 2.

→ mám fci dvou proměnných x a y

→ budu tedy hledat derivace: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y^2}, \frac{\partial^2 f}{\partial x \partial y}$

→ pokud derivuji parciálně podle x, s y nakládám jako s konstantou a opačně

pravidlo pro derivování součinu

- $$\boxed{\frac{\partial f}{\partial x}} = \frac{\partial}{\partial x}\left[(x+y^2)\,e^{x+y^2}\right] = \frac{\partial}{\partial x}(x+y^2)\cdot e^{x+y^2} + (x+y^2)\cdot\frac{\partial}{\partial x}\left(e^{x+y^2}\right) =$$

$$= 1\cdot e^{x+y^2} + (x+y^2)\cdot \underbrace{e^{x+y^2}\cdot 1}_{\text{derivace složené funkce}} =$$

$$= \boxed{e^{x+y^2}(1+x+y^2)}$$

derivace součinu

- $$\boxed{\frac{\partial f}{\partial y}} = \frac{\partial}{\partial y}\left[(x+y^2)\,e^{x+y^2}\right] = \frac{\partial}{\partial y}(x+y^2)\cdot e^{x+y^2} + (x+y^2)\cdot\frac{\partial}{\partial y}\left(e^{x+y^2}\right) =$$

$$= 2y\cdot e^{x+y^2} + (x+y^2)\cdot \underbrace{e^{x+y^2}\cdot 2y}_{\text{derivace složené fce}} =$$

$$= e^{x+y^2}\left(2y + 2y(x+y^2)\right) = \boxed{2y\,e^{x+y^2}(1+x+y^2)}$$

- $$\boxed{\frac{\partial^2 f}{\partial x^2}} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left(e^{x+y^2}(1+x+y^2)\right) =$$ opět součin

$1+y^2$ je konstanta, derivace je $= \emptyset$

$$= \frac{\partial}{\partial x}\left(e^{x+y^2}\right)\cdot(1+x+y^2) + e^{x+y^2}\cdot\frac{\partial}{\partial x}(1+x+y^2) =$$

$$= e^{x+y^2}(1+x+y^2) + e^{x+y^2}\cdot(1) = \boxed{e^{x+y^2}(2+x+y^2)}$$

- $\boxed{\dfrac{\partial^2 f}{\partial y^2}} = \dfrac{\partial}{\partial y}\left(\dfrac{\partial f}{\partial y}\right) = \dfrac{\partial}{\partial y}\left(2y\, e^{x+y^2}(1+x+y^2)\right) \overset{\text{součin tří funkcí}}{=}$

$(f\cdot g\cdot h)' = f'gh + fg'h + fgh'$

$$= \frac{\partial}{\partial y}(2y)\cdot e^{x+y^2}\cdot(1+x+y^2) + 2y\cdot\frac{\partial}{\partial y}\left(e^{x+y^2}\right)\cdot(1+x+y^2) +$$

$$+\ 2y\cdot e^{x+y^2}\cdot\frac{\partial}{\partial y}(1+x+y^2) =$$

$$= 2\cdot e^{x+y^2}\cdot(1+x+y^2) + 2y\cdot e^{x+y^2}\cdot 2y\cdot(1+x+y^2) +$$

$$+\ 2y\cdot e^{x+y^2}\cdot 2y =$$

$$= 2e^{x+y^2}\cdot\left[(1+x+y^2) + 2y^2(1+x+y^2) + 2y^2\right] =$$

$$= \boxed{2e^{x+y^2}(1+x+2xy^2+5y^2+2y^4)}$$

derivuji podle x, tj. $2y$ = konst a mohu "vytknout" před derivování

- $\boxed{\dfrac{\partial^2 f}{\partial x\partial y}} = \dfrac{\partial}{\partial x}\left(\dfrac{\partial f}{\partial y}\right) = \dfrac{\partial}{\partial x}\left(2y\, e^{x+y^2}(1+x+y^2)\right) =$

$$= 2y\cdot\left[\frac{\partial}{\partial x}\left(e^{x+y^2}(1+x+y^2)\right)\right] \overset{\text{derivace součinu}}{=}$$

$$= 2y\left[\frac{\partial}{\partial x}\left(e^{x+y^2}\right)\cdot(1+x+y^2) + e^{x+y^2}\cdot\frac{\partial}{\partial x}(1+x+y^2)\right] =$$

$$= 2y\left[e^{x+y^2}\cdot(1+x+y^2) + e^{x+y^2}\cdot 1\right] =$$

$$= \boxed{2y\, e^{x+y^2}(2+x+y^2)}$$

- OVĚŘTE, ŽE $\dfrac{\partial^2 f}{\partial y\partial x} = \dfrac{\partial^2 f}{\partial x\partial y}$